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A Spectral Shallow-water Wave Model with Nonlinear Energy- and Phase-evolution

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LONG-TERM GOALS

Our long-term goal is to provide the international community with the capability to determine the hydro-dynamic regimes of coastal environments (including large-scale catastrophic flooding) at the highest level, both operationally, with open source computer codes supported in the public domain, and scientifically with experimental open source codes.

OBJECTIVES

Numerical wave modeling in oceanic and coastal waters is usually based on a phase-averaged approach (spectral models), whereas close to shore, in the surf zone and in harbors, it is usually based on a phase-resolving approach (time domain models). In the present project we are developing a spectral wave model that is applicable to a wide variety of scales (from the deep ocean to small-scale coastal regions).

Implemented on an unstructured geographical grid covering all scales (to allow the required extreme flexibility in spatial resolution), this allows waves to propagate from the ocean, across the shelf into coastal waters, around islands, across tidal flats, through channels and over shoals, into the surf zone and into harbors, but also towards cliffs and into fjords, while fully and simultaneously accounting for all relevant processes of propagation (shoaling, refraction, diffraction, transmission and reflection), generation (by wind), dissipation (white-capping, depth-induced breaking and bottom friction) and wave-wave interaction (triad and quadruplet).

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APPROACH

Our approach is **(a)** to develop a version of our existing 3rd-generation spectral energy wave model (SWAN) on an unstructured grid and to fully integrate this version with the ADCIRC circulation model (of Notre Dame University). This allows two-way interactions between waves, wind, currents and sea level variations. The unstructured grid is common to both models, with interactions between the two models passing through this grid (the up-grading of the circulation model and its coupling with the wave model is addressed in a separate, ONR funded, project[1]); and **(b)** to expand the energy-based wave model to account for spatial coherence in the wave field. Our initial approach for the latter was to simultaneously evaluate a coupled set of equations**:** a spectral *energy* balance equation and a spectral *phase* evolution equation on the unstructured grid.

During the course of this project theoretical and practical considerations have convinced us to no longer pursue a separate phase model; instead we represent the statistics of the wave field using the coupled mode spectrum (Janssen, 2008) – which captures spatial coherence in the wave field and reduces to the energy density spectrum under quasi homogeneous conditions – and derive an evolution equation for the coupled mode spectrum from first principles. A Taylor approximation to the resulting pseudo-differential equation then serves as the governing equation of our model. The intention is to keep exploring the possibilities of this approach, and its extension to non-linear waves, after the formal completion of the project.

Leo H. Holthuijsen	Principal investigator. Associate professor at Delft University. Formulates the basic problem and approach and supervises all activities in this study. He is one of the original authors of the spectral energy model that is used in this study (SWAN).
Guus S. Stelling	Co-principal investigator. Full professor at Delft University. Supervises the development of the numerical techniques and the overall progress. Responsible for awarding Ph.D. degree to P. Smit (see below).
Marcel Zijlema	Associate professor at Delft University. Develops and implements the information technology, in particular for the unstructured grid and the coupling between the wave model and the circulation model. Supports the development and implementation of the numerical methods. Releases the final products in the public domain.
Nico Booij	Associate professor at Delft University (retired). External advisor (in a private capacity; the lead author of the original SWAN model) to support both the numerical methods and representations of physical processes involved.
Tim Janssen	Assistant Professor of Oceanography at San Francisco State University. Introduced the idea to use the Coupled mode spectrum and its approximate evolution equation as an alternative to the phase model and collaborates on its further development.
Pieter Smit	as of Nov. 1, 2008: Ph.D. student with M.Sc. degree in Civil Engineering (Delft University). Is responsible for the development, implementation

[1] Wave and circulation prediction on unstructured grids. *Joannes J. Westerink, University of Notre Dame and Clint Dawson, University of Texas at Austin and Rick A. Luettich, University of North Carolina at Chapel Hill*

and testing of the phase model and is presently developing a model based upon the coupled mode spectrum.

WORK COMPLETED

The SWAN wave <u>energy</u> model: The unstructured-grid formulation for the energy model has been coded, tested and released in the public domain (the public domain version of SWAN has been extended with this option; http://130.161.13.149/swan/download/info.htm). It has been coded such that a large-scale computation is possible in parallel on a large set of processors (parallelized code). The coupling between ADCIRC and SWAN is now fully implemented and has been validated.

The SWAN wave <u>phase</u> model: A deterministic evolution model has been formulated in terms of a spectral energy density equation and its associated spectral phase equation. This model has been implemented as an experimental finite difference code and has been shown to generate acceptable results for a set of academic cases (wave reflection in front of a vertical wall, Snel's law). For more complicated situations (submerged shoal; Chawla 1995) the model reproduces qualitatively correct results, but is unable to reproduce the measurements quantitatively.

The coupled mode spectrum: The governing equation for the coupled mode spectrum has been derived from first principles. A first order Taylor approximation of the resulting pseudo-differential equation is shown to result in the classical energy density equation. A third order Taylor approximation has been derived and it is our intention to use this approximation as the governing equation for the evolution of the coupled mode spectrum.

RESULTS

The SWAN wave <u>phase</u> model: A formulation based upon a separate evolution equation for phase and energy spectrum has been investigated to see if such an approach is feasible and can be implemented in a reasonably easy and efficient way. The phase evolution equation was postulated to be similar to the energy density equation, but now including a simple source term that accounts for the phase development due the propagation in geographical space. Futhermore, a weak coupling between the phase and energy evolution equations is introduced by the introduction of a spectral diffraction term based upon Holthuijsen (2003).

Unfortunatly, a fundamental problem with this approach appears to be the rigourous definition of the phase function in a continuous spectral formulation. From a theoretical viewpoint, the phase is defined most naturally along a wave ray from geometric optics. Extending the ray concept to a local continuous spectrum is not trivial as the concept of a phase spectral *density* function makes little sense; which is mainly due to the lack of a conservation and superposition principle for the phase.

Some of the difficulties can be avoided if only a single incident wave packet is considered. Its spectral representation will then form a narrow distribution along a principle direction θ and frequency f. Associated with the wave packet is a phase function which is constant over the frequency/direction domain occupied by the wave packet, and is undefined outside of this range. If the incident wave spectrum consist of a broad range of frequency and directions, it needs to be subdivided into wave

packets which are then simulated individually and are - in later stage - recombined to obtain the final result.

An experimental numerical implementation - based upon finite differences and using a similar solution technique as found in SWAN – has been developed and applied to several academic cases and to an experimental setup due to Chawla (1995). For simple bottom topographies (e.g. shoaling on a plane beach, specular reflection) the model performs well, even when considering large meshsizes with regard to the wave length.

The performance of the model for more complicated bottom topographies is best illustrated with results from the Chawla experiment. Figure 1a shows a snapshot of the free surface together with a ray traced solution while Figure 1b shows relative wave heights and bottom contour lines. Qualitatively the model correctly shows the cross sea pattern behind the shoal, which is absent in phase averaged spectral wave models such as SWAN.

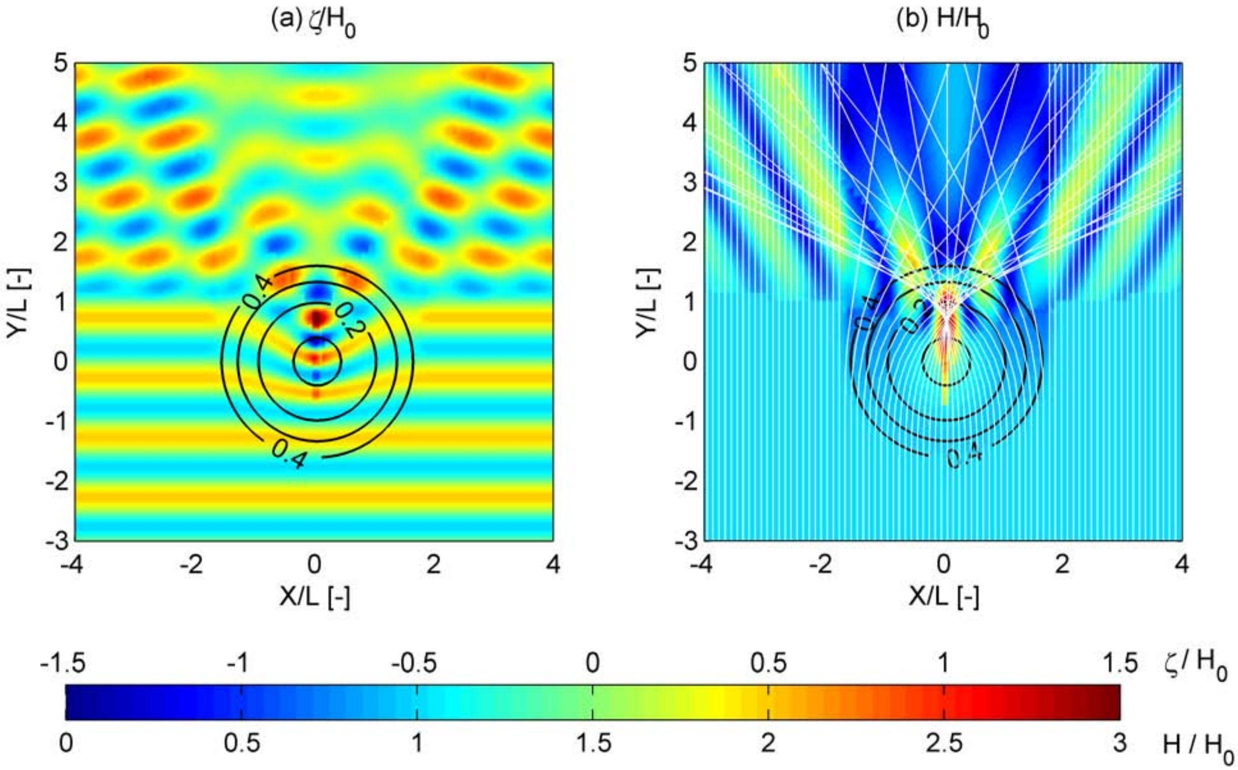

Figure 1 Monochromatic wave of 1 Hz and amplitude of 0.0118 m travelling in the positive Y direction incident on a circular shoal (Chawla, 1995). Surface elevation and wave height are scaled with the wave height at the boundary, horizontal dimensions are scaled with the wave length (L = 1.5 m). **(a)** Snapshot of the relative free surface elevation contours (colored patches), and bottom contours. (Thick black lines) **(b)** Relative wave height contours (colored patches), bottom contours (thick black lines) and wave rays obtained from a ray tracer (white lines).

However the application of the phase model to this situation is not straightforward; especially in the focal zone behind the shoal and in the neighourhood of the caustics the solution is prone to errors and stopgap measures are needed to prevent unphysical solutions. Furthermore diffraction was not accounted for in these computations as enabling it would result in unstable model behaviour. These problems all contribute to poor quantitative correspondence between measurements and model results.

Concluding we can state that the combined phase-energy model can reproduce qualitatively and quantitatively good results for simple bottom topographies. However for more complicated situations - especially when caustics occur – the model requires stopgap measures that are problem dependent and are difficult to generalize. The combination of these practical and the aformentioned theoretical problems convinced us to no longer persue a separate phase evolution model.

The coupled mode spectrum: An elegant alternative to a deterministic phase/energy spectrum representation of the wave field is the coupled mode spectrum[2] (Janssen, 2008). This spectrum forms a generalization of the well-known variance density spectrum for inhomogeneous fields and originates from quantum mechanics where it was introduced by Wigner (1932). It has the important property that it reduces to the variance density spectrum for (locally) quasi-homogeneous. However, for spatially inhomogeneous fields (e.g. standing waves) it can become negative and this generally prohibits its direct interpretation as a variance density spectrum. It is important to note that even though the coupled mode spectrum might (locally) have no direct physical interpretation, its integral parameters are always well defined; for instance, the first order moment still represents the local variance (Cohen, 1989).

The governing equation for the coupled mode spectrum in a slowly varying medium is given by a pseudo-differential equation (Bremmer, 1972; Bastiaans, 1997) that exactly describes the evolution of the coupled mode spectrum; including diffraction. Using a Taylor series approximation for the pseudo-differential operators results in an infinite order linear partial differential equation. Interestingly, if the Taylor expansion is truncated after the first order, we obtain the classical energy density equation (Bremmer, 1972). This again confirms that the coupled mode spectrum manifests itself in a similar way as the energy density spectrum for quasi-homogeneous conditions. Truncating at the third order introduces third order terms in the evolution equation that account for spatial variations in the wave field on a typical length scale in the order of a wavelength. It is this third order equation for the coupled mode spectrum that we intent to use as our governing equation.

We are presently analyzing the third order equation and are in the process of implementing the evolution equation into a numerical model. An important result thus far is that we can show that the equation can be written in conservative form, implying that the coupled mode spectrum still obeys a conservation law. Furthermore, for a constant bottom and in the absence of lateral variations, the resulting equation becomes a linearized Korteweg-de Vries type equation that can also be obtained from the classical amplitude evolution equations.

Using the coupled mode spectrum and its associated approximate third-order evolution equation adheres to our original goal to develop a single spectral wave model that is applicable on a wide variety of scales and can use can use coarse grids where the wave field varies slowly (e.g. on the ocean) and fine grids where rapid spatial oscillations are present (e.g. behind a submerged shoal). Furthermore, we hope to use the same formalism to derive an isotropic evolution equation for the bi-spectrum and using this to improve modeling of the non-linear triad wave-wave interactions.

[2] This was suggested to us by dr. T.T Janssen of San Francisco State university who also supports us in the further development.

IMPACT/APPLICATIONS

If successful, the potential future impact of the full wave model (i.e., on an unstructured grid and including spatial coherence) would be to improve the quality and the operational handling of wave modeling at all scales from oceanic waters to small-scale coastal regions, surf-zones, cliffs and harbors. SWAN$^{\varphi\text{-US}}$ would not only have superior performance in *present* applications of 3rd-generation and Boussinesq models, but also great potential for *new* applications. For instance, (future) data-adaptive unstructured grids would allow a detailed representation of small, moving atmospheric or oceanic driving forces such as hurricanes or oceanic rings. Adaptive grids would also allow high-resolution wave computations near a stationary or moving target such as a bay or an individual ship.

The coupling with circulation models such as the ADCIRC model in the present effort is equally to improve the quality and the operational handling of circulation modeling at all scales from oceanic waters to small-scale coastal regions (joint effort with Notre Dame University). The combination has shown to provide accurate computations of large-scale catastrophic flooding.

TRANSITIONS

The development in this study are aimed at acquiring a numerical wave model that provides a first step towards an operationally more accurate and user-friendly platform than the present combination of different wave models (phase-averaged and phase resolving) that is used for wave predictions in coastal regions.

The task of developing a SWAN version on an unstructured grid (SWAN^{US}) has been being carried out in close cooperation with scientists and engineers from the Notre Dame University who have coupled SWAN^{US} to their hydrodynamic model (ADCIRC) to better predict storm surges (see parallel study funded by ONR**:** Wave and circulation prediction on unstructured grids, by J. J. Westerink, University of Notre Dame, C. Dawson, University of Texas at Austin and R.A. Luettich, University of North Carolina at Chapel Hill). Delft University advised and assisted these scientists and engineers in their task to achieve this.

SWAN^{US} has been released in the public domain on the dedicated SWAN web site of the Delft University of Technology and is therefore freely available to private industry, universities and government agencies. Furthermore, the status of SWAN^{US} has moved from experimental to operational and it is now fully integrated into the latest version of SWAN which has several hundred active users.

RELATED PROJECTS

The present operational version of the SWAN model has been developed by the same group of the Delft University that is carrying out the present project, with the active support of ONR and the Dutch Ministry of Public Works. The Ministry continues to financially support the development; management and maintenance of the public domain SWAN at the Delft University and at other institutes of research and development. The circulation model that has been coupled to the new model SWAN^{US} is the ADCIRC model of Notre Dame University (USA) in a parallel ONR funded project (Wave and circulation prediction on unstructured grids, by J. J. Westerink, University of Notre Dame, C. Dawson, University of Texas at Austin and R.A. Luettich, University of North Carolina at Chapel Hill).

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